I do not understand why to choose η as done in line 16 of page 11. Let us look at the following:
We want the inequality (in line -14 of page 11,)

$$\|u - x'\|^2 (\frac{1}{2\lambda\alpha_0} - 2) \leq 1 + 2c^2 + \frac{c + \lambda}{\alpha_0}$$

which, for $1 - 4\alpha_0\lambda > 0$, is equivalent to

$$\|u - x'\|^2 \leq (\alpha_0 + 2\alpha_0 c^2 + c + \lambda)\frac{2\lambda}{1 - 4\alpha_0\lambda},$$

and this leads to make the choice in such a way that

$$(\alpha_0 + 2\alpha_0 c^2 + c + \lambda)\frac{2\lambda}{1 - 4\alpha_0\lambda} \leq \varepsilon^2. \tag{1}$$

First Possibility of choice:
The inequality (1) is equivalent to the choice of λ such that

$$2\lambda^2 + 2(\alpha_0 + 2\alpha_0 c^2 + c + 2\alpha_0\varepsilon^2)\lambda - \varepsilon^2 \leq 0, \tag{2}$$

and computing the roots (in λ) of the associated equation yields that the latter is **equivalent to**

$$\lambda \leq L := \frac{1}{2}\sqrt{K^2 + 2\varepsilon^2} - \frac{1}{2}K,$$

where $K := \alpha_0 + 2\alpha_0 c^2 + c + 2\alpha_0\varepsilon^2$. Combining this, with the above restriction $1 - 4\alpha_0\lambda > 0$ gives for η the choice

$$\eta = \min\left(\frac{1}{4\alpha_0}, L\right). \tag{3}$$

Second possibility of choice (the best one in my point of view).
Write in line 16 of page 11:
"Choose a positive real $\eta < \frac{1}{4\alpha_0}$ such that $(\alpha_0 + 2\alpha_0 c^2 + c + \eta)\frac{2\eta}{1+4\alpha_0\eta} < \varepsilon^2$."

This second possibilty of choice does not require either any computation or any verification, and it follows directly from the fact the first member in the latter inequality tends to 0 as $\eta \downarrow 0$.

So, we modify:
line 17 with: "Let $\lambda > 0$ and ..." in place of "Let $\lambda \in]0, \eta[$ and ..."
line 27: we make the choice of η as above in line 27.